Buildings are three-dimensional objects, but architectural communication about them occurs primarily in the two-dimensional medium of drawings. The use of drawings to communicate about buildings goes back to ancient times; however, the idea of basing a systematic relationship between a building’s shape and its two-dimensional representations on quantification is much more recent. Systematic here refers to the assumption that a complete two-dimensional visual representation (for instance, a set of drawings) of a three-dimensional object can be formulated by means of a clearly defined and consistently applied mathematical procedure. This procedure may be, for instance, a perspectival or orthogonal projection, but it must be consistently applicable: once the shape of the object is known, one should be able to produce its two-dimensional representations from any given side. The procedure must also enable one to depict the complete shape of the three-dimensional object using a set of two-dimensional representations—the way, for instance, modern three-dimensional computer modeling enables one to “rotate” the shape of an object on a computer screen. Finally, the procedure must not be misleading. The resulting representations must not suggest the existence of things they are not meant to represent—all points and lines in a drawing must be representations of their spatial equivalents. There should be no point or line in a drawing that does not represent some point or line in the spatial disposition of objects (real or imaginary) that the drawing represents. Fifty years ago, architecture schools taught descriptive geometry as the mathematical discipline that enabled architects to achieve such systematic representations, resolve difficult spatial relationships between elements, and develop their ability to visualize the buildings and spaces they designed. Presumably, modern three-dimensional computer modeling, which relies on the same assumption that quantification can guarantee the consistency and completeness of two-dimensional representations of three-dimensional shapes, has made this training obsolete.

In this article I will consider the first theoretical articulation of the idea of a systematic and consistent representation of three-dimensional shapes in a two-dimensional medium in the history of architecture. It should not be surprising that Leon Battista Alberti was the first to formulate the idea—or that his formulation cuts deep in some of the central assumptions that architects necessarily make about space as the medium in which they operate.

Mental Rotation 1

I will start by considering the best-known case of noncomputer three-dimensional modeling, mental rotation. Being able to see an object on a computer screen from different sides—to rotate the object—clearly parallels imagining an object from different sides, the procedure that psychologists call “mental rotation.” In other words, a picture on a computer screen can show an object only from a single side; one needs a series of images that show the object from different sides in order to be able to describe its three-dimensional properties—which is equivalent to imagining the object from
Thinking about the three-dimensional properties of objects ultimately consists in imagining them visually from various sides. The procedure fundamentally relies on the fact that the limits of human visual imagination coincide with the limits of Euclidean geometry. One can visually imagine a centaur or a physically impossible event, but one cannot visualize a geometrically impossible object, such as a pentagonal square or two points connected with more than one straight line. In this sense, too, mental and computer three-dimensional modeling are similar: no three-dimensional modeling software makes it possible to visualize on the screen a sphere whose intersection with a plane is a hexagon. There are good reasons for this. A non-Euclidean room may have, for instance, an infinite number of ceilings, all parallel to and equidistant from the floor. No builder could build it, and the number of clients who would need a computer model of such a room is unlikely to motivate commercial software firms.

It is much harder to explain why human visual imagination is subject to Euclidean geometry.\(^1\) Visual imagination and mental rotation became serious topics of psychological research in the 1970s. In an experiment that has become famous, Roger Shepard presented subjects with images of similar spatial figures (Figure 1).\(^4\) The subjects were asked to determine in which cases the images showed different compositions of cubes and in which cases they showed the same compositions rotated (seen at different angles). Most subjects solved these problems without difficulty, but for Shepard’s team the important point was how much time the subjects needed to respond. It turned out that the amount of time needed was directly proportional to the angle of rotation. The subjects needed twice as much time to respond to an image rotated by 120 degrees as they did for one rotated by 60 degrees. They also reported that they solved the problems by rotating the objects in their imaginations.\(^5\)

The important aspect of mental rotation—the one carefully imitated by various computer programs for three-dimensional computer modeling—is that it cannot yield a geometrically impossible result. Ultimately, our implicit knowledge of geometry enables us to imagine what a thing looks like from different sides; without our implicit assumptions about the geometrical properties of spatial objects we would not be able to perform mental rotation. But is this knowledge acquired or inborn? In three-dimensional modeling programs it is built into the software. About four decades ago, Stephen Kosslyn suggested that geometry is constitutive of the medium in which human visual imagination operates. He proposed treating visual imagination as equivalent to the cathode-ray tube (CRT) display used in the television sets of that time.\(^6\) Obviously, Kosslyn was not suggesting that people have TVs in their heads; rather, he was asserting that human visual imagination is comparable to a TV screen. In other words, when images represent spatial objects, the

\(^1\) Conveying the way a building smells from different sides will not get us very far; the same applies to other senses as well, and even the sense of touch is of little use when it comes to large or complex objects.\(^2\)

\(^2\) (Conveying the way a building smells from different sides will not get us very far; the same applies to other senses as well, and even the sense of touch is of little use when it comes to large or complex objects.) When considering an object’s shape, one does not think by means of scripts or the equations of analytic geometry that can be used to define the object’s shape. Rather, when such nonvisual representations are involved, one has to think in terms of their visual interpretation in order to establish what the object would be like spatially.

\(^3\) The limits of human visual imagination coincide with the limits of Euclidean geometry. One can visually imagine a centaur or a physically impossible event, but one cannot visualize a geometrically impossible object, such as a pentagonal square or two points connected with more than one straight line.

\(^4\) The geometrical properties of the building can be conveyed fully and unequivocally by means of visual images—that the geometry of visual perception can guarantee accurate communication about the geometry of physical objects.\(^2\) (Conveying the way a building smells from different sides will not get us very far; the same applies to other senses as well, and even the sense of touch is of little use when it comes to large or complex objects.) When considering an object’s shape, one does not think by means of scripts or the equations of analytic geometry that can be used to define the object’s shape. Rather, when such nonvisual representations are involved, one has to think in terms of their visual interpretation in order to establish what the object would be like spatially.

\(^5\) The important aspect of mental rotation—the one carefully imitated by various computer programs for three-dimensional computer modeling—is that it cannot yield a geometrically impossible result. Ultimately, our implicit knowledge of geometry enables us to imagine what a thing looks like from different sides; without our implicit assumptions about the geometrical properties of spatial objects we would not be able to perform mental rotation. But is this knowledge acquired or inborn? In three-dimensional modeling programs it is built into the software. About four decades ago, Stephen Kosslyn suggested that geometry is constitutive of the medium in which human visual imagination operates. He proposed treating visual imagination as equivalent to the cathode-ray tube (CRT) display used in the television sets of that time.\(^6\) Obviously, Kosslyn was not suggesting that people have TVs in their heads; rather, he was asserting that human visual imagination is comparable to a TV screen. In other words, when images represent spatial objects, the
Spatial properties of such objects (such as their dimensions) are encoded and represented in the same way as are the representations underlying the experience of seeing during perception. This heuristic metaphor (the “CRT model”) enabled Kosslyn’s team to formulate a series of hypotheses that they tested experimentally with success. Typical experiments involved measuring the time subjects needed to “scan” various images in their imaginations. Kosslyn endeavored to show that visual imagination is a genuine mode of thinking and not a mere by-product of more abstract, nonpictorial mental processing. (Arguably, the latter is the case with the rotation of a spatial object on a computer screen: what we see on the screen is but an external manifestation of the mathematical operations performed by the computer’s processor.)

If Kosslyn is right and our incapacity to visualize geometrically impossible spatial relationships derives from the visual medium in which we think spatially, then this incapacity could be inborn, hardwired.

Kosslyn’s CRT model was criticized by Zenon Pylyshyn. In Pylyshyn’s view, Kosslyn’s experiments seemed to confirm the CRT model because the subjects’ responses followed their tacit knowledge about the way human vision functions, not because the CRT model accurately described the nature of the (presumably hardwired) medium in which visual imagination occurs. “Tacit knowledge” consists of the assumptions an individual implicitly makes when solving a problem; very often it is neither conscious nor explicitly articulated. For instance, Pylyshyn argued, imagine a transparent yellow filter and a transparent blue filter side by side. Then imagine moving them slowly so that one crosses over the other. What color do you imagine you see through the two superimposed filters? The answer is green, but this is because we assume that green is seen when blue and yellow filters are superimposed on each other. Pylyshyn’s example suggests that we acquire our tacit knowledge through experience and learning; it is unlikely that our knowledge of the physical laws that determine the colors seen through light filters is inborn. However, this argument is much more convincing in regard to imagining physical processes than it is in the case of the geometrical properties of things (such as three-dimensional shapes). With physical events, our implicit beliefs indeed determine what we imagine: in the example of light filters, it is possible to assume that the blue and yellow filters are some special kind that make one see red when they are superimposed—and then to imagine seeing red through them. When our tacit assumptions about physical processes change, the way we imagine physical events changes as well. But this does not apply to the geometrical properties of things: whatever I know or believe, I cannot imagine two points connected by more than one straight line. I cannot decide to imagine a special kind of sphere whose intersection with a plane is a triangle.

Kosslyn and Pylyshyn both attempted to explain the results of Shepard’s experiments in the postbehaviorist era. In their time, Shepard’s experiments contributed significantly to the demise of behaviorist psychology. Notoriously, in the preceding decades, behaviorism in American psychology had dismissed research about visual imagination as unscientific. Shepard managed to show that quantifiable experimental research about human visual imagination is possible. It is therefore not surprising that in the popular media Shepard’s experiments are sometimes celebrated as the “discovery” of mental rotation. This claim is certainly inappropriate, since architects have known about mental rotation for a very long time. The discipline of descriptive geometry, which has been in use for centuries to formalize relationships among plans, sections, and elevations, fundamentally relies on the human ability to visualize objects from different sides. A drawing such as the one reproduced here from Andrea Palladio’s 1570 treatise I quattro libri dell’architettura would be incomprehensible if we were not able to understand it as a representation of one architectural detail shown from different sides (Figure 2). The assumption is that a set of drawings like this one provides the human mind with information that is sufficient for the viewer to form a three-dimensional mental model of the detail described, otherwise stonemasons would not be able to produce the details of the Ionic order.

Note the geometry that underlines the organization of the drawing: the planes and lines that would coincide in three-dimensional space are carefully aligned. The geometrical consistency of the spatial properties of objects is reflected in the consistent geometrical organization of their depiction. The geometrical consistency of the three-dimensional mental model (e.g., in the architect’s mind) is reflected in a consistent system of geometrical transformations that define the organization of the drawing. The fact that our visual imagination (and the mental rotations that it enables) necessarily operates according to Euclidean geometry means that geometry must underlie our every attempt to use imagination to think visually about the spatial properties of objects. Because it is impossible to achieve high levels of geometrical precision in imagination, the architect is forced to resolve and define on paper, using geometrical construction, the specific relationships among various shapes, sizes, and distances. Such geometrical resolutions are achievable because it is possible to articulate the problem mentally, in imagination. We still have to imagine visually the dispositions of geometrical bodies in order to solve geometrically, with precision, problems pertaining to their spatial relationships. Ultimately, the capacity to think spatially using visual imagination enables us to produce, understand, and use geometrically consistent two-dimensional representations of three-dimensional
objects because the geometry of visual imagination is the same as the geometry of architectural space. The question is, when did the geometry of space start to guarantee the validity of visual representations in architectural communication?

Alberti and the Geometry of Architectural Works

More than a century before Palladio published his drawing, Leon Battista Alberti wrote that a work of architecture “consists of lineaments and matter.” Interpretive debates about the meaning of Alberti’s technical term lineamenta started as early as the sixteenth century. The word occurs ninety-three times in Alberti’s architectural treatise. In all its occurrences, it can be translated into English as “shape,” and it is generally clear that Alberti meant something like “shape as (geometrically) defined by lines.” In the opening of the first book of his treatise Alberti directly explained that lineaments are immaterial and that we perceive the same lineaments on different buildings when we perceive that individual parts, their placement, and their order mutually correspond in all angles and lines. Similarly, in De statua Alberti described instruments that one can use to copy sculptures by measuring their shapes and reproducing “the lineaments and the position and collocation of parts.”

Like the English word shape, lineamenta may refer to the shape imagined in the mind of the architect, the shape

Figure 2 Detail of the Ionic order (Andrea Palladio, I quattro libri dell’architettura [Venice: Domenico de Franceschi, 1570], 34; Collection Centre Canadien d’Architecture/Canadian Centre for Architecture, Montreal).
represented by means of drawings (models), or the shape of a physical building. A well-known Aristotelian principle says that (the content of) knowledge is identical with its object; in this case, lineamenta would be the content of the architect’s thoughts, a property of a physical building, and that which is communicated by means of drawings. A physical building, as mentioned, thus consists of lineaments and matter. Lineamenta are also purely mental, the content of an architect’s idea. Alberti wrote that a lineament is a definite and constant description (perscriptio), conceived and perfected by the rational soul and the learned ingenium. Perscriptio is another of Alberti’s technical terms. The noun and its related verb perscribere occur sixteen times in De re aedificatoria. Standard Latin dictionaries recommend translating them as referring to verbal rather than visual descriptions; nevertheless, except for three occurrences, Alberti in De re aedificatoria used the term to refer to visual descriptions of formal properties. We thus read about perscriptiones of geometrical figures, parts of buildings, and fossils as representations of the shapes of animals. Out of eleven contexts in which Alberti used perscriptio to refer to a visual description, in seven he directly implied that this description was two-dimensional, while the remaining four contexts allow for such interpretation (though they do not necessarily imply it). Alberti directly contrasted the perscriptio of a building with its model because of the former’s two-dimensionality. The textual evidence thus suggests interpreting perscriptiones as two-dimensional mental representations. Alberti’s referring to perscriptiones as mental images is a quattrocento equivalent of Kosslyn’s CRT model, a mental equivalent of representation on a two-dimensional TV screen.

Ontologically, for Alberti, what an architect makes is mental content that is conveyed to workers by means of drawings and materialized in executed buildings. Mario Carpo, who uses the modern English word design for this mental content, observes that “in Alberti’s theory, the design of a building is the original, and the buildings are its copy.” He adds, “Alberti’s design process relies on a system of notation whereby all aspects of a building must be scripted by one author and unambiguously understood by all builders.” Alberti indeed assumed strong separation between the architect as the author who determines the immaterial lineaments or shapes of the building, a job that is performed in the mind, and the hand of the artisan that imposes these shapes on matter. His statement in the preface to De re aedificatoria that a building consists of lineaments and matter would have been immediately recognized by his contemporaries as a departure from the fundamental Aristotelian thesis that a material object consists of form and matter. Forma in the Latin Aristotelianism of his time is a rendering of Greek eidos: it is not the shape of an object but its essence, the what-it-is-to-be-that-thing. For instance, the essence of a dog is the dogness that it shares with other dogs, not its shape, which can vary. Aristotelian essence, morphe in Greek and forma in Latin, is instantiated in a thing as its nature; Albertian lineamentum is instantiated in a thing as its shape. For Alberti, two buildings share the same lineaments if they have the same lines and angles. The instantiation of the shape conceived by the architect is comparable to the instantiation of Aristotelian essence in matter; it is achieved by the hand of the craftsman, and if collaboration between architect and craftsman is going to be possible, the mental capacities of both must enable it. It is therefore fair to ask about the way the mental processes that enable such collaboration, thinking and communicating about spatial objects, would have been conceived of in Alberti’s time.

Imagination in Aristotelian Psychology

Alberti could not have read twentieth-century research on visual mental imagery, but Aristotelian psychology of his time would have provided him with ample support for a theory of perscriptiones as two-dimensional mental representations of three-dimensional objects. Renaissance psychology was based almost exclusively on the interpretation of and commentary on Aristotle’s De anima. Aristotle’s treatise not only had a huge circulation, additionally strengthened by the religious importance of questions about the immortality of the soul, but it was also mandatory reading in almost all European universities during the Renaissance. In fact, at the time there was no alternative description of human cognition and mental capacities. Aristotelians or not, Renaissance thinkers had to rely on the classification of these capacities presented in Aristotle’s De anima.

Imagination—and visual imagination in particular—plays a central role in Aristotle’s description of human cognition. Contrary to some twentieth-century philosophers who insisted that there can be no thinking without language, Aristotle insisted that there can be no thinking without imagination. In De anima he describes the way the percepts received by the external senses (seeing, hearing, and so on) are first assembled in common sense—“common” because it combines the information received from various external senses. Common sense formulates, for instance, the awareness that the white substance I see is also sweet. Together with memory, common sense enables imagination to form the representations of external objects called phantasmata. It is from the phantasm that the intellect subsequently extracts the essence of the object—that is, recognizes what the thing is. The nature of phantasmata is described in greater detail in De memoria than in De anima, since the latter treatise is mainly concerned with differentiating between the imagination
and other capacities of the soul. In De memoria Aristotle says that the affection produced by sensation in the soul is a picture (zographema) and that it is like an image (typos) of the thing perceived; perception is comparable to the impression of a signet ring into wax. In the next paragraph he starts his explanation of the intentionality of mental visual representations by saying, “Assuming that there is in us something like an image [typos] or a drawing [graphe].” Since the standard translation of zographema is “picture” and that of graphe is “drawing,” the implication is that phantasms are two-dimensional representations. It may be argued that the term typos, “imprint,” suggests three-dimensionality because of the reference to the impression of a signet ring into wax. (Alternatively, it is plausible that Aristotle is talking about imprinting merely in order to describe how such representations come about.) But even if typos is understood three-dimensionally, as a kind of relief, when it comes to the representation of architectural works, such reliefs would function in the same way as two-dimensional representations: a set of such representations, showing the object from different sides, would be needed to represent the three-dimensional shape of a building. One would still encounter the same problems that result from the need to define geometrically the relationship between a three-dimensional shape and a set of drawings depicting it. De memoria would have been available to Alberti in a medieval translation by James of Venice that also emphasizes the two-dimensionality of phantasms. In the medieval Latin rendering it is figura that is imprinted into wax, while the second sentence cited above discusses figura and pictura. Figura could be understood as three-dimensional, but, as mentioned, this three-dimensionality is the three-dimensionality of a relief; pictura is definitely two-dimensional. Alberti’s understanding of prescriptions as two-dimensional representations thus fits well in the mainstream of Renaissance Aristotelian psychology.

Geometrical Consistency of Two-Dimensional Representations

This is, however, also the point at which Alberti departs from Aristotle. Even in the writings that preceded De re aedificatoria he followed the long tradition of Greek, Arab, and medieval optical theorists rather than the Aristotelian account of human visuality. There were indeed good reasons to avoid Aristotle’s account of human vision. In Aristotle’s view, light is the actualization of the medium of vision—which precludes the existence of light rays that travel in straight lines. The problem with this view is that it makes it impossible to explain why we perceive things where they are, their relative positions, and their spatial relationships to each other. For instance, if a chair is to the left of a table, I perceive it there because the light rays that reach my eyes from the chair arrive from a position to the left of that from which the light rays from the table reach my eyes. For this explanation to work, one must postulate straight light rays traveling from the object to the eye. But Aristotle’s explanation leaves no space for light rays—and, as a result, Aristotelian optics de facto cannot explain why things are perceived where they are. There is nothing in Aristotle’s account that would ensure that I perceive the chair to the left of the table, if it is really there. Such a theory of vision is certainly unsatisfactory, especially for Alberti, who concentrated his efforts on defining a geometry-based pictorial account of the shapes and spatial distances between objects. In classical antiquity, we find a comprehensive mathematical treatment of human vision in Euclid’s Optics. Starting from the assumption that light rays are straight lines, Euclid described how the perceived size of an object (the size of its visual angle) depends on its distance from the viewer. This was basically the geometrical version of the central problem that the early Renaissance theorists of perspective endeavored to resolve: the relationship between the distance of an object from the viewer and the object’s perceived size. However, Euclid, as a mathematician, never contemplated, and was probably not interested in, the implications of his theorems for the visual depiction of objects by means of drawings. He mathematically articulated the way the perceived size of an object changes with distance, but he did not bother to ask how his mathematical description could be applied in drawing. In simple words, Aristotle talked about images but conceived of them as independent of the geometry of light; Euclid described the geometry of light but had no interest in images.

The pictorial application of the geometry of human visuality to visual representations is Alberti’s important topic in De pictura. The starting point of his account is the observation that every surface can be defined through the specification of the size, position, and length of lines. The statement directly contradicts Aristotle’s view that we primarily perceive colors and, consequently, surfaces. For Alberti, surfaces are defined by lines. He then proceeds by observing that light rays travel in straight lines; since they connect the eye and the object, the perception of every line on the object can be analyzed geometrically—it can be defined by means of a triangle whose base is the line mentioned and the opposing point of which is the human eye. Taken together, such triangles make up the pyramid of sight. This reasoning implies that the totality of human visual experience can be described geometrically. A picture is a section through the pyramid of sight—it is a plane that shows what we would see through a window located at the place of the picture plane. In other words, there is the eye and there is the object depicted in perspective; the light rays that reach the eye from the object travel in straight lines. One could place a window between the object and the eye and draw on the glass the
object as one sees it through the glass. Perspective is the procedure that generates the same drawing by geometric means. A perspectival drawing therefore delivers to the eye a bundle of light rays equivalent to the one that would reach the eye from the depicted object. Thus understood, perspectival geometry describes the totality of visuospatial perceptual experience. In the little essay *Elementa picturae* Alberti states that there is nothing in nature that can be perceived by the eyes and yet not represented in lines. At the same time, the geometrical procedure for the construction of perspectival drawings that he describes can represent any shape from any side, and it does not allow that there could be a part of a drawing that would not geometrically define something in the disposition of objects depicted. As Ernst Gombrich has pointed out, “The first consequence of the [Albertian] window idea is that we cannot conceive of any spot on the panel which is not significant, which does not represent something.” This corresponds to the idea of systematic two-dimensional representations of three-dimensional objects (whose completeness and consistency are enabled by the mathematical procedures that define them) described in the opening of this article.

The central point of Alberti’s writings about the visual arts is the quantification of spatial relationships and visual experience; *De pictura, De statua, Elementa picturae, and Descriptio urbis Romae de facto* all endeavor to show that there are no spatial relationships that cannot be arithmetically or geometrically compared. According to *De pictura*, light guarantees the geometrical nature of human vision: light rays, which make human visual experience possible, are the equivalents of straight lines. Ultimately, those properties that define lineaments are the same properties (the shapes of buildings) that are depicted using light rays. The geometry of physical shapes is translatable fully and without residue into the geometry of two-dimensional visual representations, *perscriptiones* drawn or imagined. *De pictura* is mainly concerned with the geometrical construction of perspectival *perscriptiones*, but there is ultimately no reason plans, sections, or elevations would be excluded. Alberti’s important point is that all visible properties of objects are geometrically definable as sets of lines; the sets of lines that define the shape of an object are its lineaments, and a building consists of lineaments and matter. The geometry of light rays guarantees the geometry of two-dimensional representations of three-dimensional objects. We have seen that this would not work with Aristotelian understanding of light—and one can similarly compare Alberti’s understanding of human vision with the theory presented in Lucretius’s *De rerum natura*, which became available in Florence as a result of Poggio Bracciolini’s discovery in 1417. According to Lucretius, objects constantly emit their own images (*simulacra*), which are like membranes drawn from their outermost surfaces that move swiftly through the air. They are broken when they run into wood or rocks; they pass through glass and are reflected by mirrors. The theory leaves unclear how we can perceive the distance between objects (which is immaterial and therefore emits no image) and leaves no possibility for a quantified description of visual space, such as perspective. Alberti’s *perscriptiones*, however, are definable by means of the geometry of light rays—and since they are two-dimensional, we need more than one of them to depict a three-dimensional object from all its sides. To achieve this, we need to rotate the object in our minds or using drawings, the way most architects today would on a computer screen. So we are back to mental rotation.

### Mental Rotation 2

The nature of the cognitive processes that enable mental rotation was Alberti’s additional difficulty with Aristotelian psychology. On one hand, he had reason to endorse Aristotelian account of the importance of visual imagination—his *perscriptiones* are close approximations of Aristotle’s *phantasmata*, and at the time there was no alternative cognitive psychology to explain visual imagination anyway. We have seen that for Aristotle, the lower, material, and perishable strata of the human soul consisted of the five external senses (vision, smell, hearing, taste, and touch) and the internal senses (the common sense, imagination, and memory) that processed the information received through the external senses. In the debate between Kosslyn and Pylyshyn, Aristotle would side with Kosslyn, given that in *De memoria* he explicitly states that the products of imagination, phantasms, are stored in memory. In other words, our memory stores two-dimensional images as two-dimensional images—it does not encode them in some other way. For Aristotle, there is no thinking without imagination. Unlike Pylyshyn, he does not conceive of imagination as a mere by-product of more abstract, nonpictorial mental processing. It is from phantasms that the intellect extracts the essence of the thing, recognizes what the thing is.

On the other hand, any attempt to explain mental rotation within the framework of Aristotelian cognitive psychology must run into serious problems. Insofar as *zographema* is indeed a two-dimensional representation, its geometrical relationship to the (shape of the) objects it represents is unclear in the Aristotelian account. For Aristotle, we have seen, the geometry of light does not provide a direct (geometrical) connection between the object and its representation. Consequently, if no geometrical account of visuospatial representation is possible, it is unclear how the geometrical properties of objects can be preserved in the process of (mental) representation. And if they are not preserved in representation, it is even less clear how they can
be articulated consistently in mental rotation. An even more fundamental problem is that, according to Aristotle, mathematical thinking is, ultimately, a job of the intellect—a stratum of the soul above the imagination. If mental rotation were to produce geometrically accurate results in the imagination, the imagination would have to follow the instructions it received from the intellect. In Aristotle’s cognitive scheme, however, the intellect extracts the essence of the thing from the phantasm, but there is no account of any (mathematical) feedback that it may provide to the imagination. It is consequently not clear how imagination can solve the geometrical problems that arise from imagining a three-dimensional object from different sides in a series of two-dimensional phantasms.

Alberti does not have a solution for this problem, and, arguably, he does not need to have one. His treatises pertain to visual arts and architecture, not cognitive psychology. But his use of specific terms (and avoidance of others) shows that he was aware of the problem. The standard Aristotelian technical terms that could lead to the articulation of the problem, such as intellectus or imaginatio, do not appear once in the De re aedificatoria. Three crucial psychological terms, or mental capacities, are mentioned in this work: mens, animus, and ingenium. The last of these is the capacity for invention, the ability to create new things. Spatial thinking, including the ability to interpret two-dimensional perscriptions three-dimensionally, happens, Alberti says, in animo et mente. The two terms seem to be interchangeable in his writing. In the Aristotelian psychological tradition, animus and mens are the most general names for the rational capacities of the soul and pertain to the totality of the human rational soul. In the context of Alberti’s architectural treatise, animus refers to the cognitive soul in general—it is not limited to the intellect but also includes imagination, common sense, and memory. It thus comprises both the intellect and the lower, perishable, strata of the rational soul. Mens is used the same way, but not so often. The two terms are often used conjointly: Alberti says that we imagine buildings in animo et mente and that we can contemplate forms independent of matter in animo et mente. Alberti, one may surmise, was aware of the problem, and he avoided it by using the most general Aristotelian terms that refer to the rational soul, without committing himself to the particularities of the Aristotelian psychological account.

Epilogue

A discussion of the possibilities (or limits) of the use of geometry in the representation of architectural works is ultimately a discussion about the nature of space and the nature of architectural works. Alberti articulated first (most directly in Elementa picturae, as mentioned) the important thesis that the content of every possible visual experience, seeing or imagining a thing, is quantifiable and geometrically describable. The principle seems obvious enough to us today—but we have seen that this was not necessarily always the case, and it was certainly not the case with Aristotle. Alberti’s principle that the totality of human visual experience is quantifiable is the fundamental assumption of architectural practice. This principle makes three-dimensional computer modeling software possible. One can only try to imagine what architectural work would be like today if it were impossible to describe by means of geometry the shapes of buildings as one perceives them.

In other words, without Alberti’s principle, there would be no modern architectural practice. It should be noted, however, that the principle comes prepackaged with a strong claim on universality: everywhere where light rays are straight lines the geometry of human vision is going to be the same. As Alberti himself put it, parts of a sculpture can be reproduced at different places and subsequently put together as long as the system of measurement used is the same across the locations. Alberti’s theory of visual representation relies on assumptions about space that Ernst Cassirer and Erwin Panofsky have called homogeneity: it is a space in which geometry functions the same everywhere and in which it is possible to draw identical geometrical figures in every direction. Since Panofsky suggested, almost a century ago, that the early Renaissance understanding of space as homogeneous enabled the discovery of the geometrical construction of perspective, it has often been asserted that space was not conceived as homogeneous before the quattrocento. Some more radical authors have even suggested that we do not inhabit homogeneous space and that we are merely culturally conditioned to think that we do. Considering Cassirer’s and Panofsky’s definition of homogeneous space, such theses are hard to defend: in a non-homogeneous space it is not possible to draw the same figure from every point in every direction. A figure can be a simple line (or a rectangle whose width is zero), and claiming that the people of the past could not have conceived of space as homogeneous boils down to saying that they did not assume that they could draw the same line from point A to point B and from B to A. Had architects started understanding space as homogeneous only in the Renaissance, before the Renaissance they would not have known that the length of a wall must be the same regardless of the end of the wall from which it is measured. This was not the case. What changed at the beginning of the Renaissance was not the understanding of space. Rather, the Italian urban environments of the era were characterized by close social interactions between artisans and intellectuals that made it possible for them to articulate dilemmas pertaining to visual representation as mathematical problems and resolve them in ways that were consistent.
with geometrical knowledge available from many centuries earlier. Alberti’s and Brunelleschi’s great innovation was not the introduction of the assumption of the homogeneity of space but the systematic application of the geometrical nature of space to the problem of two-dimensional representation of three-dimensional objects. And it was an immensely influential innovation, judging by the way it dominates modern architectural practice.

**Notes**

1. My special gratitude to Mario Carpo for stimulating my interest in the problems discussed in this article during our conversation at the SAH conference in Richmond, Virginia, in 2002. The article reconstructs the Renaissance Aristotelian perspective on those aspects of Alberti’s theoretical views that, as Carpo has described, have significant parallels with modern architectural practice. I would also like to express my gratitude to Victor Caston, Peter Lautner, Mark Gage, Peter McPherson, and Bojan Tepavčević for help and advice in the preparation of the article and to Cameron Moore for help with the written English of the article.


5. Subsequent experiments have shown that pigeons can be trained to solve the same problems as successfully as humans, but in the pigeons’ case there is no correlation between the angle of rotation and the time needed for response. This has led researchers to conclude that pigeons do not employ mental rotation when solving such problems. Valerie D. Holland and Juan D. Delius, “Rotational Invariance in Visual Pattern Recognition by Pigeons and Humans,” *Science* 218 (1982), 804–6.


12. For instance, in order to define geometrically the intersection line of two prisms, we have to imagine how these prisms would intersect with specific planes from different sides.


15. See ibid., 177–83, for a complete survey of all the contexts in which this term appears in *De re aedificatoria*.

16. “Neque habet lineamentum in se, ut materiam sequatur, sed est huiusmodi, ut eadem plurimis in aedificis esse lineamenta sentiatuis, ubi una atque eadem in ills spectetur forma, hoc est, ubi eorum partes et partium singularum situs inter se conveniant totis angulis totisque lineis.” Alberti, *De re aedificatoria*, 19–21 (4.30–4.10).


18. Alberti also says that *lineamenta* are constitutive of the discipline of architecture, the *res aedificatoria*: “Tota res aedificatoria lineamentis et structura constituta est.” Alberti, *De re aedificatoria*, 19 (4.30).


20. “Haec cum ita sint, erit ergo lineamentum certa constansque perscriptio concepta animo, facta lineis et angulis perfectaque animo et ingenio erudito.” Alberti, *De re aedificatoria*, 21 (4v.13–14). Insofar as *perscriptiones* are two-dimensional (as argued later in this article), this sentence may seem to contradict the understanding of *lineamenta* as three-dimensional shapes. In fact, Alberti’s “haec cum ita sint” pertains to the previous sentence, where he explains, “Et licet integras formas praeserciere animo et mente seclusa omni materia.” Ibid., 21 (4v.10–11). Lineaments thus can be contemplated in the mind independent of the matter that instantiates them, and when they are contemplated in the mind, they are represented in the
form of two-dimensional perscriptio. Similar understanding is proposed by the Russian commentator Vasily Pavlovich Zubov, who translates this section as “формы зданий могут быть мысленно вложены без всякой матери. . . Если это так, то очертание есть нечто определенное и постоянный план, задуманный в уме, образуемый линиями и углами и выполняемым духом и умом совершенного.” Leon Battista Alberti, De descritcis o dolositce (Desyat knig o zodchestve) (De re aedificatoria), 2 vols., trans. (vol. 1) and commentary (vol. 2) by Vasily Pavlovich Zubov (Moscow: Vesnyuznaya Akademiya Architekturi 1935-37), 1:12. Zubov thus translates perscriptio as “plan” (маяд) and therefore understands it as two-dimensional, while in the commentary (2:279) he warns that lineamenta are three-dimensional.

21. See the systematic survey of Alberti’s use of these terms in Mitrovic, Serene Greed, 37–38, 183–85.

22. Ibid., 183–85.


24. The translation by James of Venice states at 450a29 “passionem factam per sensum in anima . . . velut picturam quadrat,” at 450b30 “factum enim mos tum velut figuram quandam sensibilis sitc sigillantes annulis,” and at 450b16 “si est simile sicut figura aut pictura in nobis huius ipsius sensus.” Cited according to Thomas Aquinas, In Aristotelis libros De sensu et sensato De memoria et remissencia commentarium, ed. Raymond Spiazzi (Turin: Marietti 1973), 95. See also Aristotelii libros anmes ad animalium cognitionem attentius cum Averros Cordubiensis variis in eosdem commentariis (Venice: Apud Luncttas, 1562), vol. 6, part 2, 18v and 18r.

25. Aristotle’s main account of human vision is in De anima, 418a27–419b3. Aristotle bases his account on the existence of the transparent medium (“τον θαλασσων διαφανον”); ibid., 418b3. Light is then the actualization of the transparent medium as transparent (“φως δε ποννη η τοτοιον ενεργον του διαφανους διαφανους”); ibid., 418b9. It would be wrong to say that light travels, Aristotle says; such movements could escape our observation in a small intervening space, but not at such great distances as the ultimate points in the East and West. Ibid., 418b21–26. See also Aristotle, De sensa, 438a27, 446b27. However, there are sections in Aristotle that contradict this account of visual perception; see David Lindberg, *Theories of Vision from Al-Kindi to Kepler* (Chicago: Chicago University Press, 1976), 217n39.

26. As Lindberg points out, “For example, how, on the Aristotelian theory, can individual parts of the visual field be distinguished? If colored bodies produce qualitative changes in all parts of a transparent medium to which they have rectilinear access, how do we see one object here and another object over there? What is the nature or source of the directional capabilities of sight? Aristotle’s theory of vision provides no answer to these questions.” Lindberg, *Theories of Vision*, 58–59.

27. As C. D. Brownson notes, “The Optics is primarily associated with the concern to find a mathematical formulation of the laws of sight, while linear perspective has been employed only for the construction of pictures.” The important difference is that “Euclid’s Optics studies the apparent size, shape, and position of objects from a point of observation, while the central problem for linear perspective is determining the relative size, shape, and placement of objects in a scene as they appear at a picture plane.” C. D. Brownson, “Euclid’s Optics and Its Compatibility with Linear Perspective,” *Archive for History of Exact Sciences* 26 (1982), 188, 165.


29. This is what psychologists call proximal perception. See Hershcenson, *Visual Space Perception*, 118.


31. Alberti, *De anima*, 431a17, 431b3, 432a8; Aristotle, *De memoria*, 449b49. For a survey of the view that thinking is inseparable from language, see Michael Loosnitz, *Linguistic Turns in Modern Philosophy* (Cambridge: Cambridge University Press, 2006).
48. “Visum per triangula fieri cuius basis visa quantitas cuiusae laterae sunt iidem ipsi radii qui a punctis quantitatis ad oculum protenduntur.” Alberti, De pictura, 1.6.
49. Ibid., 1.7.
50. Ibid., 1.19.
51. “Ex his quae sequentur, omnis ratio et via perscribendi componendique lineae et angulos et superficies explicabitur notaque reddetur adeo ut nihil in rerum natura sit, quod ipsum oculis posit perspicui, quin id hinc instructus perfacile possit lineis perfinire atque exprimere.” What follows (“qua sequentur”) is a list of twenty-five simple geometrical constructions. Alberti, Elementa picturarum, E.
55. Ibid., 4.26–44, 4.143–45.
56. Ibid., 4.98, 4.150–54.
57. Aristotle, De memoria, 451a2–12.
58. For a survey and discussion of the use of these terms in De re aedificatoria, see Mitrović, Serene Greed, 153–57, 201–8.
60. See Mitrović, Serene Greed, 147–64, 201–8.
61. Alberti, De re aedificatoria, 835 (173v.7–9), 859 (174v.16), 861 (174v.25).
62. Ibid., 11 (3.18), 21 (4v.10). Alberti says that one often cannot refrain “quin mente et animo aliquas aedificationes commentemur.”
63. Alberti, De statua, 6, 16.
64. Such geometrical homogeneity does not preclude the possibility that space may not be homogeneous in some other sense—for example, that heavy bodies strive to move in one direction and light bodies in another. For Panofsky’s definition of the homogeneity of space, see Erwin Panofsky, Die Perspektive als symbolische Form, in Deutschsprachige Aufsätze, ed. Karen Michels and Martin Warnke (Berlin: Akademie Verlag, 1998), 2: 664–757, 2:667–68. For a history of the debate about the homogeneity of space, see Branko Mitrović, “Leon Battista Alberti and the Homogeneity of Space,” JSAH 63, no. 4 (2004), 424–39.
65. Panofsky argued that perspective as a method of visual communication is merely a social convention that arose in the Renaissance and that space was not understood as homogeneous before the Renaissance. He noted, first, that the human retina is semispherical while the laws of perspective are defined for a plane—ergo, these laws cannot describe the real image that light creates on the retina. However, the geometrical construction of a perspectival drawing is not meant to replicate the retinal image; rather, as Alberti himself described in De pictura, it is meant to generate the same drawing as the one that would be drawn on a glass plane placed between—the eye and the object depicted. The form of the retina is ultimately irrelevant; the aim of a perspectival drawing is to deliver a bundle of light rays equivalent to the one that would reach the eye from the depicted object. Second, Panofsky argued that the ancient Greeks perceived differently from modern men, since the geometrical postulates of Euclid’s Optics differ from those necessary for the geometrical construction of perspective. In other words, Panofsky claimed that the geometry of sight that was described by Euclid was particular to ancient Greeks and was incompatible with modern visuality that arose in the Renaissance. C. D. Brownson has shown that this claim was based on Panofsky’s misunderstanding of Euclid’s Greek terminology. Brownson, “Euclid’s Optics,” 181–85. The eighth theorem of Euclid’s Optics states, “Τὰ ἤντε ἀναλόγους διαστήματα ἀπὸ τὸν ὅμοιον ὁμοίως ὑπὸ ὁμοίωσιν ὁμότατα.” Panofsky failed to understand the mathematical, technical meaning of Euclid’s ἀναλόγους. As Brownson points out, the mathematical meaning is “having the same or constant ratio,” while Panofsky understood it simply as a relationship in which one quantity has a dependent relation to another.